ASPECTS OF COSMOLOGICAL RELATIVITY

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ABSTRACT

In this paper we review the *cosmological relativity*, a new special theory of relativity that was recently developed for cosmology, and discuss in detail some of its aspects. We recall that in this theory it is assumed that gravitation is negligible. Under this assumption, the receding velocities of galaxies and the distances between them in the Hubble expansion are united into a four-dimensional pseudo-Euclidean manifold, similarly to space and time in ordinary special relativity. The Hubble law is assumed and is written in an invariant way that enables one to derive a four-dimensional transformation which is similar to the Lorentz transformation. The parameter in the new transformation is the ratio between the cosmic time to the Hubble time (in which the cosmic time is measured backward with respect to the present time). Accordingly, the new transformation relates physical quantities at different cosmic times in the limit of weak or negligible gravitation.

The transformation is then applied to the problem of the expansion of the Universe at the very early stage when gravity was negligible and thus the transformation is applicable. We calculate the ratio of the volumes of the Universe at two different times T_1 and T_2 after the Big Bang. Under the assumptions that $T_2 - T_1 \approx 10^{-32}$ sec and $T_2 \ll 1$ sec, we find that $V_2/V_1 = 10^{-16}/\sqrt{T_1}$. For $T_1 \approx 10^{-132}$ sec we obtain $V_2/V_1 \approx 10^{50}$. This result conforms with the standard inflationary universe theory, but now it is obtained without assuming that the Universe is propelled by antigravity.

New applications of the theory are presented. This includes a new law for the decay of radioactive materials, that was recently developed by Carmeli and Malin. The new law is a modification of the standard exponential formula, when cosmic times are considered instead of the ordinary local times. We also show that there is no need to assume the existence of galaxy dark matter; the Tully-Fisher law is derived from our theory. A significant extension of the theory to cosmology that was recently made by Krori, Pathak, Das and Purkayastha is

given. In this way cosmological relativity becomes a general theory of relativity in seven dimensions of curved space-time-velocity. The solutions of the field equations in seven dimensions obtained by Krori *et al.* are given and compared to those of the standard Friedmann-Robertson-Walker. A completely new picture of the expanding Universe is thus obtained and compared to the FRW one.

1 Introduction

Few important problems in cosmology are widely discussed these days. The first is the problem of dark matter, its theory and its experimental verification. This problem is intimately related to the amount of matter in the Universe, or more accurately to the value of $\Omega = \rho/\rho_c$ where ρ_c is the critical matter density and ρ is the actual matter density in the Universe.

A second problem is that of the inflation of the Universe at the very early stage, at which time the Universe expanded drastically. This problem is related to particle physics. What are the reasons for the inflation? Was there a kind of antigravity? A third problem is the age of the Universe. If one determines the age of the Universe by nuclear synthesis measurements of the Earth or our galaxy, and compares it with that obtained from measurements of the Hubble constant (using a certain model for the Universe), these two ages are not exactly equal. Also the problem of directly measuring the Hubble constant seems to depend on the distance scale of the galaxies used for the measurements.

In this lecture we address ourselves to the problem of the inflation at the early stage of the Universe. At that time gravitation was in no existance. Within this assumption of negligible gravitation we develop a theory which enables us to discuss and obtain some exact results that standard methods are unable to provide.

Some new applications of the theory are presented. This includes a new law for the decay of radioactive materials, that was recently developed by Carmeli and Malin. The new law is a modification of the standard exponential formula, when cosmic times are considered instead of the ordinary local times. We also discuss the problem of galaxy dark matter. We show that there is no need to assume the existence of dark matter for galaxies. It is shown that the Tully-Fisher law can be derived from our theory. A significant extension of the theory to cosmology that was recently made by Krori *et al.* is given. In this way cosmological relativity becomes a general theory of relativity in seven dimensions of curved space-time-velocity. The solutions of the field equations in seven dimensions obtained by Krori are given and compared to those of the standard Friedmann-Robertson-Walker. A completely new picture of the expanding Universe is thus obtained and compared to the FRW one.

2 Consequences of the Hubble Expansion

The Hubble law expresses the simple relationship between the receding velocities of galaxies to their distances, thus gives a mathematical expression to the observation that the Universe is expanding. This is an experimental fact, in which underlies the assumption that the observed redshift in the spectrum emitted from galaxies is due to Doppler effect. In mathematical terms the Hubble law is given by

$$\mathbf{v} = H_0 \mathbf{R}. \tag{1}$$

H_0 is the Hubble constant. In reality H_0 is not a constant in cosmic times; and that is due to gravity whose effect varies as the Universe expands. In the limiting case of negligible gravitation assumed in this lecture, H_0 can be considered to be a constant and does not depend on the cosmic time.

If we denote by $\tau = H_0^{-1}$ the Hubble time, then τ can be considered as the age of the Universe in this particular case of neglecting gravity. We write the Hubble law in the trivially different form

$$\mathbf{R} = \tau \mathbf{v}, \tag{2}$$

in order to compare it with the well-known expression for the propagation of light, $R = ct$. In Eqs. (1) and (2) $\mathbf{R} = (x, y, z)$. Equation (2) can thus be expressed as

$$x^2 + y^2 + z^2 = \tau^2 v^2, \tag{3}$$

where $\mathbf{v}$ is the outgoing velocity. Accordingly

$$x^2 + y^2 + z^2 - \tau^2 v^2 = 0. \tag{4}$$

It will furthermore be assumed that a relationship of the form (4) holds at any cosmic time t. Accordingly, if we denote distances and velocities at two different cosmic times t and t' by x, y, z, v and x', y', z', v', then it will be assumed that

$$x'^2 + y'^2 + z'^2 - \tau^2 v'^2 = x^2 + y^2 + z^2 - \tau^2 v^2. \tag{5}$$

Equation (5) resambles that for the propagation of light viewed from two different inertial frames of references moving with a constant velocity with respect to each other. In our case we have what might be called cosmic frames of references which differ from each other by a cosmic time.

3 The Cosmological Transformation

The question now arises as to what is the transformation between the four variables x, y, z, v and x', y', z', v' that leaves unaffected the invariance equation (5)?

For simplicity it will be assumed that $y' = y, z' = z$, thus we have

$$x'^2 - \tau^2 v'^2 = x^2 - \tau^2 v^2. \tag{6}$$

What transformation keeps the last formula invariant? The solution of Eq. (6) can be written as

$$x' = x \cosh\psi - \tau v \sinh\psi, \qquad \tau v' = \tau v \cosh\psi - x \sinh\psi.$$

At $x' = 0$ we have $\tanh\psi = x/\tau v = t/\tau$. As a result we have

$$\sinh\psi = \frac{t/\tau}{\sqrt{1 - \frac{t^2}{\tau^2}}}, \qquad \cosh\psi = \frac{1}{\sqrt{1 - \frac{t^2}{\tau^2}}}.$$

Consequently the transformation is given by

$$x' = \frac{x - tv}{\sqrt{1 - \frac{t^2}{\tau^2}}}, \qquad v' = \frac{v - \frac{tz}{\tau^2}}{\sqrt{1 - \frac{t^2}{\tau^2}}}, \qquad y' = y, z' = z. \tag{7}$$

Here t is the cosmic time measured with respect to us, now, and goes backward. The transformation (7) is called *the cosmological transformation*.

4 Cosmological Special Relativity

The transformation (7) very much resembles the well-known Lorentz transformation. In fact, one can give a formal foundation to establish a *cosmological special relativity* of the four-dimensional continuum of the three-dimensional Euclidean space and the outgoing radial velocity. We here mention only two consequences of the cosmological transformation, and for more applications and further details of such a theory the reader is referred to the author's book and earlier papers [1-7].

4.1 The Law of Addition of Cosmic Times

As is accepted nowadays, intervals of cosmic times can be added linearly. The cosmological transformation (7), however, tells us a different thing. A cosmological event that occured at the cosmic time t_1 (measured backward with respect to us) that preceded by a second event which occured before the first one at a cosmic time t_2, then the second event would appear to occur with respect to us at a backward time t_{12} given by

$$t_{12} = \frac{t_1 + t_2}{1 + \frac{t_1 t_2}{\tau^2}} \quad (\textit{cosmic times addition law}). \tag{8}$$

This law of addition of cosmic times can be tested by applying it to the decay cosmic times occured in the radioactive materials in determining the ages of our Earth and our galaxy.

4.2 Inflation at the Early Universe

At the early Universe gravity was completely negligible, and thus the cosmological transformation (7) may be applied. To this end we proceed as follows.

The line element for the Universe is given by

$$\tau^2 dv^2 - \left(dx^2 + dy^2 + dz^2\right) = ds^2. \tag{9}$$

Hence one has

$$\tau^2 \left(\frac{dv}{ds}\right)^2 - \left[\left(\frac{dx}{dv}\right)^2 + \left(\frac{dy}{dv}\right)^2 + \left(\frac{dz}{dv}\right)^2\right]\left(\frac{dv}{ds}\right)^2 = 1, \tag{10}$$

or

$$\left(\tau^2 - t^2\right)\left(\frac{dv}{ds}\right)^2 = 1. \tag{11}$$

Multiplying the last equation by ρ_0^2, where ρ_0 is the matter density of the Universe at the present time, we obtain for the matter density at a backward cosmic time t

$$\rho = \tau \rho_0 \frac{dv}{ds} = \frac{\rho_0}{\sqrt{1 - \frac{t^2}{\tau^2}}}. \tag{12}$$

Since the volume of the Universe is inversely proportional to its density, it follows that the ratio of the volumes at two cosmic times t_1 and t_2 with respect to us (we choose $t_2 < t_1$) is given by

$$\frac{V_2}{V_1} = \sqrt{\frac{1 - \frac{t_2^2}{\tau^2}}{1 - \frac{t_1^2}{\tau^2}}} = \sqrt{\frac{(\tau - t_2)(\tau + t_2)}{(\tau - t_1)(\tau + t_1)}}. \tag{13}$$

For cosmic times t_1 and t_2 very close to the Hubble time τ, we may assume that $\tau + t_2 \approx \tau + t_1 \approx 2\tau$. Accordingly

$$\frac{V_2}{V_1} \approx \sqrt{\frac{\tau - t_2}{\tau - t_1}}. \tag{14}$$

Denoting now by $T_1 = \tau - t_1$ and $T_2 = \tau - t_2$, with $T_2 > T_1$. T_1 and T_2 are the cosmic times as measured from the Big Bang. We thus have

$$\frac{V_2}{V_1} \approx \sqrt{\frac{T_2}{T_1}}. \tag{15}$$

For $T_2 - T_1 \approx 10^{-32}$ sec and $T_2 \ll 1$ sec, we have

$$\frac{V_2}{V_1} \approx \sqrt{\frac{T_2}{T_1}} \approx \sqrt{\frac{T_1 + 10^{-32}}{T_1}} = \sqrt{1 + \frac{10^{-32}}{T_1}} \approx \frac{10^{-16}}{\sqrt{T_1}}. \tag{16}$$

For $T_1 \approx 10^{-132}$ sec we obtain

$$\frac{V_2}{V_1} \approx \frac{10^{-16}}{10^{-66}} = 10^{50}. \tag{17}$$

This result conforms with the inflationary universe theory of Guth [8] and Linde [9] without assuming any model (such as the Universe is propelled by antigravity).

5 Decay Law of Radioactive Material in Cosmology

In this section we derive, following Carmeli and Malin [10], a new cosmological law for the decay of radioactive material.

We assume that the probability of disintegration during any interval of *cosmic* time dt' is a constant. Thus

$$\frac{dN}{dt'} = -\frac{1}{T}N, \tag{18}$$

where T is the lifetime of the material. (Throughout this section the time parameters t and t' will not be backward as are considered in the previous sections.)

Now, let us substitute in the formula for the addition of cosmic times, Eq.(8), $t_1 = t = t'$ (present time), $t_2 = dt$, then $t_{1+2} = t + dt'$, and making an approximation, using the fact that tdt/τ^2 is much smaller than 1, we obtain

$$t + dt' = (t + dt)(1 - \frac{tdt}{\tau^2}) = t + (1 - \frac{t^2}{\tau^2})dt. \tag{19}$$

If dt is a time interval measured by a clock, and we want to obtain dN/dt, we need to find dt'/dt from Eq.(19) and substitute it in Eq. (18). We then obtain

$$\frac{dt'}{dt} = 1 - \frac{t^2}{\tau^2}. \tag{20}$$

Equations (18) and (20) subsequently yield

$$\frac{dN}{dt'} = \frac{dN}{dt}\frac{dt}{dt'} = -\frac{1}{T}N, \tag{21}$$

or, using Eq.(20),

$$\frac{dN}{N} = -\frac{1}{T}\frac{dt'}{dt}dt = -\frac{1}{T}\left(1 - \frac{t^2}{\tau^2}\right)dt. \tag{22}$$

Integration of the last equation then gives

$$N(t) = N_0 \exp\left[-\frac{1}{T}\left(1 - \frac{t^2}{3\tau^2}\right)t\right]. \tag{23}$$

Now, when we say that t and t' are present time, we start them at $t = t' = 0$, which is the time when $N = N_0$. Eq.(23) will provide large deviations from Eq.(18) when T is comparable to τ, and we measure radioactivity over astronomical times. It is not clear how such measurements/observations can be carried out. However, it may be possible to detect minute deviations from linearity in a graph of $\ln N$ vs. t in very accurate laboratory measurements.

In principle, it follows from Eq.(23) that $N(t)$ for a given t is less than the traditional formula predicts. Namely, the material decays faster that expected.

6 Galaxy Dark Matter as a Property of Spacetime

In this section we generalize cosmological relativity to curved space. This will enable us to introduce gravitation.

We first describe, following Carmeli [11], the motion of a star in a central field of a galaxy in an expanding universe. Use is made of a double expansion in $1/c$ and $1/\tau$. In the lowest approximation the rotational velocity of the star will be shown to satisfy $v^4 = \frac{2}{3}GMcH_0$, where G is Newton's gravitational constant and M is the mass of the galaxy. This formula satisfies observations of stars moving in spiral and elliptical galaxies, and is in accordance with the Tully-Fisher law [12,13].

The problem of motion in general relativity is a very old one and started with Einstein and Grommer [14] who showed that the equations of motion follow from the Einstein field equations rather than have to be postulated independently as in electrodynamics. This is a consequence of the nonlinearity of the field equations and the Bianchi identities. Much work was done since then and the problem of motion in the gravitational field of an isolated system is well understood these days [15-26].

The topic of motion in an expanding universe is of considerable importance in astronomy since stars moving in spiral and elliptical galaxies show serious deviation from Newtonian gravity and the latter follows from general relativity theory [27]. It follows that the Hubble expansion imposes an extra constraint on the motion – the usual assumptions made in deriving Newtonian gravity from general relativity are not sufficient in an expanding universe. The star is

not isolated from the "flow" of matter in the universe. When this is taken into account, along with Newton's gravity, the result is a motion which satisfies a different law from the one determining the planetary motion in the solar system.

6.1 Geodesic Equation

The equation that describes the motion of a simple particle is the geodesic equation. It is a direct result of the Einstein field equation $G_{\mu\nu} = \kappa T_{\mu\nu} (\kappa = 8\pi G/c^4)$. The restricted Bianchi identities $\nabla_\nu G^{\mu\nu} \equiv 0$ implies the covariant conservation law $\nabla_\nu T^{\mu\nu} = 0$. When volume-integrated, the latter yields the geodesic equation. To obtain the Newtonian gravity it is sufficient to assume the approximate forms for the metric: $g_{00} = 1 + 2\phi/c^2$, $g_{0k} = 0$ and $g_{kl} = -\delta_{kl}$, where $k, l = 1, 2, 3$, and ϕ a function that is determined by the Einstein field equations. In the lowest approximation in $1/c$ one then has

$$\frac{d^2x^k}{dt^2} = -\frac{\partial\phi}{\partial x^k}, \tag{24}$$

$$\nabla^2\phi = 4\pi G\rho, \tag{25}$$

where ρ is the mass density. For a central body M one then has $\phi = -GM/R$ and Eq.(25) yields, for circular motion, the first integral

$$v^2 = GM/R, \tag{26}$$

where v is the rotational velocity of the particle.

6.2 Hubble's Law in Curved Space

The Hubble law was given by Eq.(1) and recasted in the form of Eq.(4) when gravity was neglected. Gravitation, however, does not permit global linear relations like Eq.(4) and the latter has to be adopted to curved space. To this end one has to modify Eq.(4) to the differential form and to adjust it to curved space. The generalization of Eq.(4) is, accordingly,

$$ds^2 = g'_{\mu\nu} dx^\mu dx^\nu = 0, \tag{27}$$

with $x^0 = \tau v$. Since the universe expands radially (it is assumed to be homogeneous and isotropic), it is convenient to use spherical coordinates $x^k = (R, \theta, \phi)$ and thus $d\theta = d\phi = 0$. We are still entitled to adopt coordinate conditions, which we choose as $g'_{0k} = 0$ and $g'_{11} = {g'_{00}}^{-1}$. Equation (5) reduces to

$$\frac{dR}{dv} = \tau g'_{00}. \tag{28}$$

This is Hubble's law taking into account gravitation, and hence dilation and curvature. When gravity is negligible, $g'_{00} \approx 1$ thus $dR/dv = \tau$ and by integration, $R = \tau v$ or $v = H_0 R$ when the initial conditions are chosen appropriately.

6.3 Phase Space

As is seen, the Hubble expansion causes constraints on the structure of the universe which is expressed in the phase space of distances and velocities, exactly the observables. The question arises: What field equations the metric tensor $g'_{\mu\nu}$ satisfies? We *postulate* that $g'_{\mu\nu}$ satisfies the Einstein field equations in the phase space, $G'_{\mu\nu} = KT'_{\mu\nu}$, with $K = 8\pi k/\tau^4$, and $k = G\tau^2/c^2$. Accordingly, in cosmology one has to work in both the real space and in the phase space. Particles follow geodesics of both spaces (in both cases they are consequences of the Bianchi identities). For a spherical solution in the phase space, similarly to the situation in the real space, we have in the lowest approximation in $1/\tau$ the following: $g'_{00} = 1 + 2\psi/\tau^2$, $g'_{0k} = 0$ and $g'_{kl} = -\delta_{kl}$, wuth $\nabla^2\psi = 4\pi k\rho$. For a spherical solution we have $\psi = -kM/R$ and the geodesic equation yields

$$\frac{d^2x^k}{dv^2} = -\frac{\partial\psi}{\partial x^k}, \tag{29}$$

with the first integral

$$\left(\frac{dR}{dv}\right)^2 = \frac{kM}{R}, \tag{30}$$

for a rotational motion. Integration of Eq.(30) then gives

$$R = \left(\frac{3}{2}\right)^{2/3} (kM)^{1/3}\, v^{2/3}. \tag{31}$$

Inserting this value of R in Eq.(26) we obtain

$$v^4 = \frac{2}{3}GMcH_0. \tag{32}$$

6.4 Galaxy Dark Matter

The equation of motion (32) has a direct relevance to the problem of the existence of the galaxy dark matter. As is well known, observations show that the fourth power of the rotational velocity of stars in some galaxies is proportional to the luminousity of the galaxy (Tully-Fisher law), $v^4 \propto L$. Since the luminousity, by turn, is proportional to the mass M of the galaxy, $L \propto M$, it follows that $v^4 \propto M$, independent of the radial distance of the star from the center of the galaxy, and in violation to Newtonian gravity. Here came the idea of galaxy dark matter or, alternatively, modification of Newton's gravity in an expanding universe.

We have seen how a careful application of general relativity theory and cosmological relativity give an answer to the problem of motion of stars in galaxies in an expanding universe. If Einstein's general relativity theory is valid, then it appears that the galaxy halo dark matter is a property of spacetime

and not some physical material. The situation resembles that existed at the beginning of the century with respect to the problem of the advance of the perihelion of the planet Mercury which general relativity showed that it was a property of spacetime (curvature).

7 Carmeli's Cosmology

Based on the full group of transformations of cosmological relativity in the seven-dimensional space of space-time-velocity, Krori, Pathak, Das and Purkayastha [28] have extended the flat-space metric to describe what the authors call Carmeli's cosmology for the expanding universe. The properties of this new cosmology were discussed in detail and compared with the standard FRW cosmology. The following is based on the paper by Krori *et al*, and for the full details the reader is referred to the original paper.

7.1 Carmeli's Cosmological Metric

The starting point was the flat-space Carmeli's metric

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2) + \tau^2(dv_x^2 + dv_y^2 + dv_z^2). \tag{33}$$

The metric (33) was subsequently extended to the following form:

$$ds^2 = c^2dt^2 - R^2(t)(dx_1^2 + dx_2^2 + dx_3^2) + T^2(t)(dv_1^2 + dv_2^2 + dv_3^2). \tag{34}$$

Here $R(t)$ is the three-space scale factor and $T(t) = H^{-1}(t)$, where $H(t)$ is the Hubble parameter.

7.2 Field Equations

The energy-momentum tensor components are given by

$$T^0{}_0 = \rho c^2, \quad T^1{}_1 = T^2{}_2 = T^3{}_3 = -p, \quad \text{and } T^\mu{}_\nu = 0;\ \mu, \nu \geq 4. \tag{35}$$

From Eqs.(34) and (35), using Einstein's field equations in seven dimensions, we obtain

$$\frac{\dot{R}^2}{R^2} + \frac{\dot{T}^2}{T} + \frac{3\dot{R}\dot{T}}{RT} = \frac{8\pi c^4\rho}{3}, \tag{36}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{3\ddot{T}}{T} + \frac{3\dot{T}^2}{T^2} + \frac{6\dot{R}\dot{T}}{RT} = -8\pi c^2 p, \tag{37}$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{2\ddot{T}}{T} + \frac{\dot{T}^2}{T^2} + \frac{6\dot{R}\dot{T}}{RT} = 0, \tag{38}$$

where a dot denotes differentiation with t. We thus have three equations for the four unknown variables R, T, ρ and p.

7.3 Solution of the Field Equations

One assumes a solution of the form $R = R_0 t^m$, where m is a positive parameter, and putting $\dot{T} = Tu(t)$, we obtain

$$t^2\dot{u} + \frac{3}{2}t^2u^2 + 3mtu + \frac{3}{2}m(2m-1) = 0. \tag{39}$$

We next define a function $v(t)$ such that $2\dot{v} - 3uv = 0$, Eq.(39) will then have the form

$$t^2\ddot{v} + 3mt\dot{v} + \frac{9}{4}m(2m-1)v = 0. \tag{40}$$

Equation (40) admits a solution of the form $v = v_0 t^n$, where v_0 is a constant and n is given by

$$n = \frac{1 - 3m \pm \sqrt{3m + 1 - 9m^2}}{2}. \tag{41}$$

From the above we obtain $u = 2n/3t$ and thus $T = T_0 t^{2n/3}$, where T_0 is a constant. Now, using the expressions of R and T in Eqs.(36) and (37) we finally obtain

$$8\pi c^4 \rho t^2 = 3m^2 + 4n^2/3 + 6mn, \tag{42}$$

$$8\pi c^2 p t^2 = -3m^2 + 2m - 8n^2/3 + 2n - 4mn. \tag{43}$$

Equations (42) and (43) provide a complete solution of the field equations.

7.4 Properties of Carmeli's Cosmology

The properties of Carmeli's cosmology are now discussed and compared with the FRW cosmology [29].

(a) $m = \frac{1}{2}$ corresponds to radiation era for both the cosmologies. At this value of m, $n = 0$ in Carmeli cosmology.

(b) In Carmeli cosmology, m varies from 1/3 to $(1+\sqrt{5})/6$ with $\rho c^2/p$ correspondingly decreasing from ∞ (dust) to 1.6191714. On the other hand, in FRW cosmology, m varies from 1/3 to 2/3 with $\rho c^2/p$ correspondingly increasing from 1 (stiff matter) to ∞ (dust). Obviously stiff matter is not admissible in Carmeli cosmology.

(c) In Carmeli cosmology, as m increases from 1/3 to $(1+\sqrt{5})/6$, p increases from 0, reaches a maximum and then falls to a certain non-zero value.

(d) In FRW cosmology, the Hubble parameter, $H = \dot{R}/R$, decreases with the passage of time for all non-zero values of m. On the other hand in Carmeli cosmology, the Hubble parameter, $H = T^{-1}$, decreases with the passage of time for $m < 1/2$ (with n positive), increases with the passage of time for $m > 1/2$ (with n negative) but is constant for $m = 1/2$ (with $n = 0$).

(e) In FRW cosmology, in the present (dust) era ($m = 2/3$), the Hubble parameter H_{FRW} and the age of the Universe t_{FRW} are related by the formula

$$H_{FRW} = \frac{2}{3}t_{FRW}^{-1}.$$

In Carmeli's cosmology, since $T = H^{-1}$ the Hubble parameter H_c and the age of the present (dust) universe, t_c, are related by

$$H_c = \frac{1}{T_0} t^{-2n/3}.$$

Following FRW cosmology, if we take $1/T_0$ to be a factor of order 1, then with $H_c = H_{FRW}$ in the present era, we have $t_c \sim t_{FRW}^{3/2n}$, i.e. $t_c \sim t_{FRW}^{3}$, since $n = 1/2$.

If this is so, then the age of the universe is very much higher according to Carmeli cosmology. Further, H_c decreases more slowly than H_{FRW} with the passage of time in the present era.

(f) The expression for cosmological redshift is [30]

$$1 + z = \frac{R(t_0)}{R(t_1)},$$

where t_0 is the epoch at which light emitted at an earlier epoch t_1 from a distant galaxy is received by our galaxy. Now, if t_0 and t_1 are assumed to be the same in both the cosmologies, then for FRW cosmology (present era)

$$1 + z_{FRW} = \left(\frac{t_0}{t_1}\right)^{2/3},$$

since $m = 2/3$, and for Carmeli cosmology (present era)

$$1 + z_c = \left(\frac{t_0}{t_1}\right)^{1/3},$$

since $m = 1/3$.

Obviously, $z_{FRW} > z_c$. In other words, redshift is less red in Carmeli cosmology.

(g) The expression for the angular size of a distant galaxy is [30]

$$\Delta\theta = \frac{d}{r_1 R\,(t_1)} = \frac{d\,(1+z)}{r_1 R\,(t_0)}, \tag{44}$$

where d is the breadth of the distant galaxy, r_1 is its coordinate distance from our galaxy, t_0 is the epoch of observation (in our galaxy) of the light emitted at the earlier epoch, t_1, by the distant galaxy and z is the redshift. If r_1 and t_1 are assumed to be same for both cosmologies, then, from Eq.(44),

$$\Delta\theta_c > \Delta\theta_{FRW},$$

since $m = 2/3$ for FRW cosmology and $m = 1/3$ for Carmeli cosmology at the present (dust) era.

(h) The expression for luminosity distance of a distant galaxy is

$$D = r_1 R(t_0)(1+z),$$

where r_1 is the coordinate distance of the distant galaxy and t_0 is the epoch at which light is received by our galaxy from the distant galaxy. Since, as already seen, above,

$$R_{FRW}(t_0) > R_c(t_0)$$

and

$$z_{FRW} > z_c.$$

We find that

$$D_{FRW} > D_c.$$

It may be noted in time that Carmeli cosmology is theoretically interesting in its own right. However, in the context of its significant deviations from the standard FRW cosmology, sophisticated observational techniques will have to be devised to assess the worth and validity of this new cosmology.

8 Appendix: Table of Numerical Results

We list below numerical results as given by Krori *et al.* [28].

m	n	$8\pi\rho c^4 t^2$	$8\pi p c^2 t^2$	$\rho c^2/p$
1/3	1/2	5/3	0	∞
0.4	0.3358898	1.4365645	0.1534974	9.3588849
0.458	0.1616129	1.1082291	0.244209	4.5380355
0.474	0.1051945	0.9879554	0.2554032	3.8682186
1/2	0	0.7500000	0.2500000	3.000000
0.514	-0.0543697	0.6288532	0.2305139	2.7280484
0.53	-0.1706014	0.3389937	0.161592	2.0978371
$\left(1+\sqrt{5}\right)/6$	$-\left(\sqrt{5}-1\right)/4$	0.00099664	0.000615572	1.6191714

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